EXAMINATION QUESTIONS/SOLUTIONS 2014-2015 Course

**Comp245**

Question

1.

Marks & seen/unseen Parts

seen ⇓

(i) (c). seen sim. ⇓

(ii) (a)

Let R denote relevant, W denote chosen words present P(R|W) = P(W|R)P(R) /P(W)

= 1 ∗ 0.01 1 ∗ + 0.01

0.1 ∗ 0.99 = 0.092

(iii) (a). np = 3 and np(1 − p) = 2, so 1 − p = 2/3 ie p = 1/3 and n = 9.

(iv) (c).

(v) (f).Denoting T the lifetime of the system, and T1 and T2 as the two independent lifetimes

with exponential distributions with parameter λ, P(T < t) = P(T1 < t ∩ T2 < t) = P(T1 < t)P(T2 < t) = (1 − e−λt)2 So the survivor function, P(T > t) = 1 − (1 − e−λt)2 = 2e−λt − e−2λt

Each 4 marks

Setter’s initials Checker’s initials Page number

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Question

2.

Marks & seen/unseen Parts

seen sim. ⇓ (i) (a)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| k | 1 | 2 | 3 | 4 | 5 |
| f(k) | 1/9 | 2/9 | 3/9 | 2/9 | 1/9 |

f(k)

3/9 62/9

1/9

- 0 1 2 3 4 5 k

6F(k) 11/9

- 0 1 2 3 4 5 k

5 marks

(b) E(X) = 1 ∗ 1/9 + 2 ∗ 2/9 + 3 ∗ 3/9 + 4 ∗ 2/9 + 5 ∗ 1/9 = 27/9 = 3

2 marks

(ii) (a) Table of expected counts

k frequency Ok expected freq Ek 1 25 100\*1/9=11.1 2 35 100\*2/9=22.2 3 21 100\*3/9=33.3 4 13 100\*2/9=22.2 5 6 100\*1/9=11.1 3 marks

(b) H0 : Observed counts equal expected counts 2 marks

(c)

X2 = ∑5i=1 (Ok − Ek)2

Ek ∼ χ24

under the null hypothesis.

X2 = (25 − 11.1 11.1)2

+ (35 − 22.2 22.2)2

+ (21 − 33.3 33.3)2

+ (13 − 22.2 22.2)2

+ (6 − 11.1 11.1)2

= 35.5

The p-value is the probability of observing these data, or data more extreme than these, if the null hypothesis is true. This is very small, <0.005 from χ2 table. 6 marks

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2.

Marks & seen/unseen Parts

seen sim. ⇓ (ii)(d) Reject the null hypothesis at both 5% and 1% significance. The model does not

fit the data well. 2 marks

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3.

Marks & seen/unseen Parts

seen ⇓ (i) (a) likelihood function for (μ, σ2) for all of the data is

L(μ, σ2) =

∏nfX(xi) i=1

= (2πσ2)− n2 expSetter’s initials Checker’s initials Page number {−∑ni=1(xi − μ)2

2σ2

}.

The log likelihood is

l(μ, σ2) = −n2 log(2π) − nlog(σ) −

∑ni=1(xi − 2σ2 μ)2 .

For MLE for μ, take partial derivative wrt μ and set this equal to zero and solve for ˆμ.

0 = ∂μ∂l(μ, σ2)|μ= ˆμ =

∑ni=1(xi − ˆμ) σ2

⇐⇒ 0 =

∑n∑i=1(xi − ˆμ) =

ni=1 xi n = ̄x.

To check this is a maximum, look at the second derivative.

∂μ∂2

2l(μ, σ2) = −nσ2 ,

which is negative everywhere, so ̄x is the MLE for μ, independently from the value of σ2. 7 marks (b)

E(X) = E(n

1∑ni=1

xi − nˆμ ⇐⇒ ˆμ =

∑ni=1

Xi) = n

1∑i=1

nE(Xi) = n1nμ = μ.

Hence X is an unbiased estimator of μ. (ii) (a) Samplemean ̄x = 1.175, s = 1.514 where s2 = 1n−1

∑ni=1(xi − ̄x)2.

3 marks seen sim. ⇓ (b) Assuming the time differences are normally distributed. Use a t distribution with 4 marks 7 df for the confidence interval.

1.175 ± t7,0.975 ∗ 1.514/√8 = 1.175 ± 2.36 ∗ 1.514/√8 = (−0.088,2.438)

3 marks (c) The 95% confidence interval includes 0, so the p-value for a two-sided test is >0.05. At 5% significance, we have insufficient evidence to reject the null hypothesis that the population mean time difference is zero. 3 marks

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Question

4.

Marks & seen/unseen Parts

seen ⇓ (i) For f to be a valid probability density function,

I. ∫ f(x) ∞≥ 0,∀x ∈ R; II. f(x)dx = 1. x=−∞ ∫ 1(ii) From part (i),

2 marks

seen sim. ⇓ x=0 Setter’s initials Checker’s initials Page number

f(x)dx = 1. So

∫ 1x=0 a + bx2dx = [ax + bx3 3]10 = a + b/3 = 1 And E(X) = 38 ⇒

∫ 1x=0 x(a + bx2)dx = 38 ⇒ [ax2 2+ bx4 4]10 = 2 a+ 4 b= 38 Solving gives a = 3/2 and b = −3/2

4 marks (iii) Var(X) = E(X2) − E(X)2

E(X2) = 32

∫ 1x=0 x2(1 − x2)dx = 32[x3 3− x5 5]10 = 32(13 − 15) = 315 = 15 So Var(X) = 15 − (38)2 = 64 5 ∗ − 64 45

= 320 19

= 0.059 4 marks

(iv) For 0 ≤ x ≤ 1F(x) =

∫ xu=0 f(u)du =

∫ xu=0

32(1 − u2)du = 32[u − u3 3]x0 = 3x − x3

2 For x < 0, F(x) = 0 and for x > 1, F(x) = 1

6F(x)

1...

...... ... .. 1/2..

.......0 median(X) ?

1 - x

4 marks

(v) (a) P(Y ≤ y) = P(3X − 1 ≤ y) = P(3X ≤ y + 1) = P(X ≤ 12 Then, ∗ (3 ∗ fY(y) (y + 3 = 1

) dFdy − Y

(y = + 3 12 1

− )3) 181= (y y + + 2 1)1

2 − where 54 1∗ (y + 1)3 −1 ≤ y ≤ 2. . .........

..y + 3 1 ) = FX(y + 3 .. .....1

) =

4 marks

(b) E(Y) = E(3X − 1) = 3E(X) − 1 = 3 ∗ 3/8 − 1 = 1/8.

Var(Y) = Var(3X − 1) = 32 ∗ Var(X) = 9 ∗ 19/320 = 171 320 = 0.53. 2 marks

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